→ Look at all the problems
→ See what you can do
→ try to solve as much as you can for 15 mins

STA 119 FINAL Review Session

*For attendance, scan the **QR code** on the **Right***

Problem 1: A company tested the lifespan of 10 randomly selected batteries, recording an average time to failure of 9.5 hours with a standard deviation of 1.4 hours.

a) Determine a 95% confidence interval for the population mean time to failure.

$\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$

$[8.499, \; 10.501]$

b) Determine a 99% confidence interval for the population mean time to failure.

$[8.061, \; 10.939]$

Problem 2: Company A claims that their light bulbs outlast the light bulbs of Company B by more than 10 days. Data from both companies has been collected and summarized below. Test the claim at the .05 level, assuming normality and equal variances.

$\Rightarrow H_0: \mu_A - \mu_B = 10$
$\Rightarrow H_a: \mu_A - \mu_B > 10$

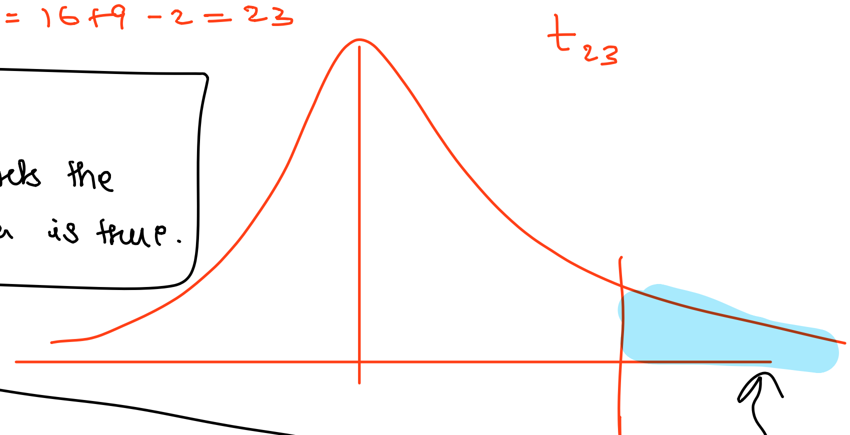
Company A: $n = 9 \quad \bar{x} = 250 \quad s = 6$

Company B: $n = 16 \quad \bar{x} = 265 \quad s = 5$

(a) Calculate $\boldsymbol{Sp}$ & Test the claim:

$df = 16 + 9 - 2 = 23$

t_{23}



1.714

$\Rightarrow \; s_p = 5.3689$

$t = 2.235$

Reject H_0:
The data supports the claim that H_a is true.

(b) Determine a 95% confidence interval:

$1 - \alpha = 0.95$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$t_{23, \alpha/2} = 2.069$

$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$265 - 250 \pm 2.069 \times 5.3689 \sqrt{\frac{1}{9} + \frac{1}{16}}$

$\Rightarrow \boxed{15 \pm 4.6284}$

$\Rightarrow$ $H_0: \mu = 500$

$H_a: \mu > 500$

Problem 3: A manufacturer claims that the average weight of a cereal box is 500g. A random sample of 16 boxes has a mean of 508g. Assume the population standard deviation is known to be 10g.
A test statistic of z = 1.6 is calculated.

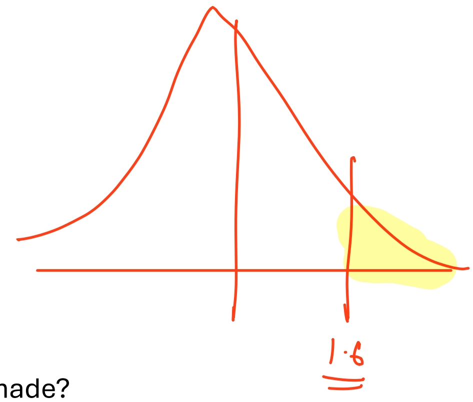
μ , n , μ_{Act} , σ

Significance level = 0.05

a) Find the p-value.

$$\Rightarrow \quad 1 - \Phi(1.6) = 1 - 0.9452$$
$$= 0.0548 > 0.05$$

Fail to Reject



b) If the true mean is actually 500g, was a Type I or Type II error made?

No error

$$P\left(Z < Z_\alpha - \frac{\mu_T - \mu_0}{\sigma/\sqrt{n}}\right)$$

Practise : $\rightarrow$ P (Type II Error) = $\Rightarrow \Phi\left(Z_\alpha - \frac{\mu_T - \mu_0}{\sigma/\sqrt{n}}\right)$

$$\Phi(1.645 - 3.2) = \Phi(-1.555) = 0.05999 \Rightarrow \Phi\left(1.645 - \frac{508 - 500}{\frac{10}{4}}\right)$$

Problem 4: A government official wishes to know what percent of the adult population favors a new gun control law. A referendum is going to be voted on in the next election. For the gun control measure to pass, more than 60% of the voters must vote for it. A survey of 2000 adults is performed. Of the 2000 adults surveyed, 1232 favor this gun control law. Can the government official be sure that the law will pass at the .05 level?

6

9

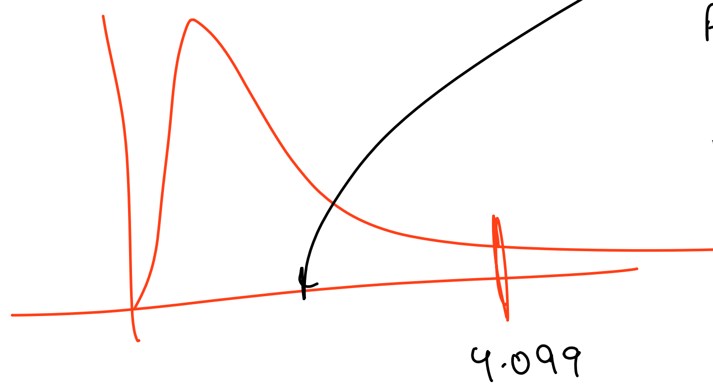
Problem 5: A professor is investigating whether two different grading methods lead to different variability in student test scores. A sample of 10 students graded using Method A has a standard deviation of 4.1 points. A sample of 7 students graded using Method B has a standard deviation of 3.3 points. Test at the 0.05 level of significance whether the variances in scores differ between the two grading methods.

$H_0: \sigma_A = \sigma_B$

$H_a: \sigma_A \neq \sigma_B$

$$F = \frac{(S_1)^2}{(S_2)^2} = \frac{(4.1)^2}{(3.3)^2} = 1.5436$$

Fail to Reject H_0



Problem 6: A researcher claims that the average reaction time for a specific task is 2.5 seconds. A sample of 10 individuals gives a sample mean of 2.7 seconds. Assume the population standard deviation is 0.4 seconds, and the z-score is 1.58.

↳ significance level = 0.05

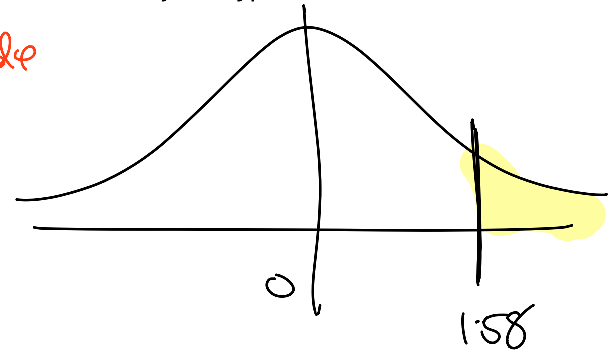
a) What is the p-value for the test?

b) If the true mean is actually 2.5 seconds, was an error made? If so, identify the type of error.

↳ no error was made

$H_0: \mu = 2.5$

$H_a: \mu > 2.5$



$$1 - \Phi(1.58) = 1 - 0.9429$$

$\approx 0.0571 > 0.05 \Rightarrow$ Fail to Reject H_0

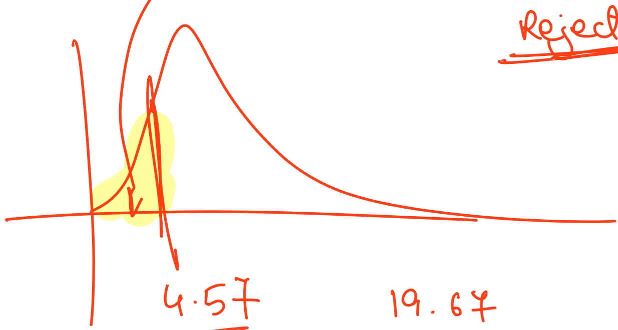
Problem 7: Company A claims that their product is more consistent than the competition, Company B. We have been collecting data from Company B for many years. We feel certain that, $\mu_b = 20.1$ *and* $\sigma_B = .12$. We now collect12 pieces of data from Company A and get $\bar{x} = 19.9$ *and* $s = .07$. Perform a .05 level test

$$\chi^2 = \frac{(n-1)\,s^2}{\sigma^2} = 2.74$$

Ho: $\sigma_A = 0.12$
Ha: $\sigma_A < 0.12$

Ho: $\sigma_A = 0.12$
Ha : $\sigma_A \neq 0.12$

Reject Ho



4.57 19.67

Problem 8: A 6-sided die is suspected of being unfair. According to theory, each face should appear with equal probability (1/6). The die is rolled 120 times with the following results:

Face	1	2	3	4	5	6
Frequency	15	22	20	18	25	20

At the 0.05 significance level, test whether the die is fair.

Ho: Dice is fair
Ha: Dice is no faire.

O

e 20 20 20 - - -

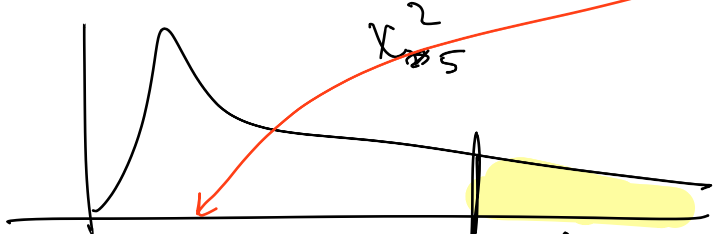
$$\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i} = \frac{\cancel{(20-15)^2}}{20} + \frac{\cancel{(20-22)^2}}{20} + \frac{\cancel{(20-20)^2}}{20} + \frac{\cancel{(20-18)^2}}{20} + \frac{\cancel{(20-25)^2}}{20} + \frac{\cancel{(20-20)^2}}{20}$$

25 4 0 4 25 0

$df = 6 - 1 = 5$

$$= \frac{25+25+4+4}{20} = \frac{\cancel{58}\;29}{\cancel{20}\;10} \Rightarrow 2.9$$

χ^2_{5}



11.07

⇒ Fail to Reject Ho
⇒ The data supports the claim that Ha is not true

Problem 9: An engineer is evaluating the consistency of two suppliers' components. Supplier A's sample of 20 parts shows a standard deviation in length of 0.8 mm, while Supplier B's sample of 18 parts shows a standard deviation of 1.1 mm.
Use a 0.01 significance level to test if the variability in component lengths differs.

Problem 10: A company claims that their candy colors are distributed as: 30% red, 25% green, 25% yellow, and 20% blue. A sample of 100 candies has the following observed counts:

Color	Red	Green	Yellow	Blue
Count	28	30	25	17

Test at the 0.05 level whether the observed distribution differs from the claimed distribution.

Problem 11:

A clinical researcher is testing H_0: $\mu = 100$ vs. H_a: $\mu \neq 100$, with a known population standard deviation of $\sigma = 15$. They want to detect a difference of $D = 0.5$ units with:

- Power = 0.85 at $\alpha = 0.05$

How large a sample is needed?

29

Problem 12: A political analyst believes that voter support for four parties is distributed as: Party A - 35%, Party B - 25%, Party C - 25%, Party D - 15%. In a random poll of 200 voters, the support was:

Party	A	B	C	D
Count	80	45	50	25
e	70	50	50	30

Ho: Expected values

Ha: ≠ expected valu

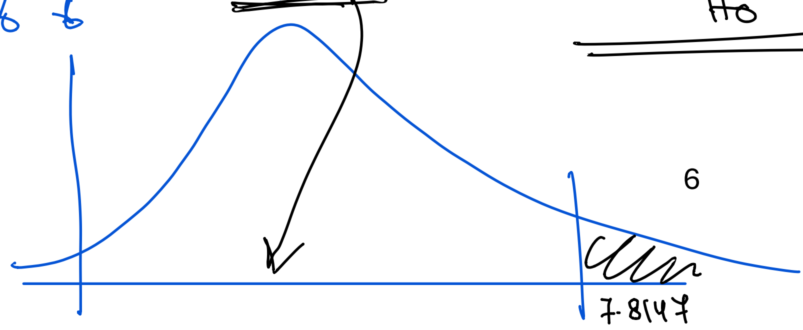
Use a 0.05 significance level to test whether the observed data matches the expected distribution.

$$\chi^2 = \frac{(70-80)^2}{70} + \frac{(50-45)^2}{50} + \cancel{\frac{(50-50)^2}{50}} + \frac{(30-25)^2}{80}$$

$$= \frac{100}{70} + \frac{25}{50}\left(\tfrac{1}{2}\right) + \frac{25}{30}\left(\tfrac{5}{6}\right) = 2.7619$$

Fail to Reject Ho

df = 3



Problem 13: A company wants to detect a change in average customer satisfaction score, originally assumed to be $\mu = 4.0$, with a standard deviation of 0.6. They test: $H_0: \mu = 4.0$ vs. $H_a: \mu \neq 4.0$
They want:

- Power = 0.90 to detect a difference of D = 0.3
- Significance level: $\alpha = 0.01$

What is the required sample size?

Problem 14: For each of the following sample sizes, determine a 95% confidence interval for the population proportion if the observed proportion ($\hat{p}$) is 0.75.

a) n = 500

b) n = 750

c) n = 1000

Problem 15: A candidate in the next election takes a poll to determine if he is losing ground in the election. In the last poll, it was determined that he had 59% of the vote. In this poll of 1000 voters, 568 said that they will vote for the candidate. Perform a .05 level test

Website for all the worksheets and their solutions.

Again, for attendance, scan this QR code: